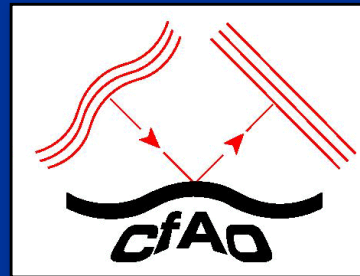


Lecture 6

Part 1: Effect of image motion on image quality

Part 2: Detectors and signal to noise ratio

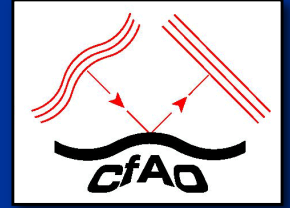


Claire Max

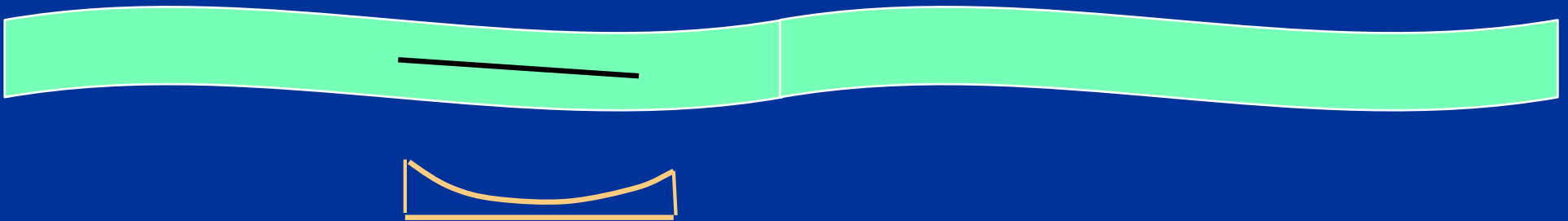
Astro 289, UC Santa Cruz

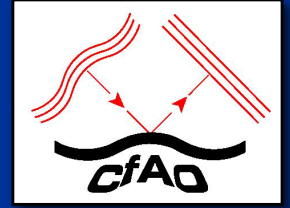
January 28, 2020

Part One: Image motion and its effects on Strehl ratio



- Sources of image motion:
 - Telescope shake due to wind buffeting (hard to model *a priori* - depends on telescope, dome, ...)
 - Atmospheric turbulence
- Image motion due to turbulence:
 - Sensitive to atm. inhomogenities $>$ telescope diam. D
 - Hence reduced if “outer scale” of turbulence is $\leq D$





Long exposures, no AO correction

$$FWHM(\lambda) = 0.98 \frac{\lambda}{r_0}$$

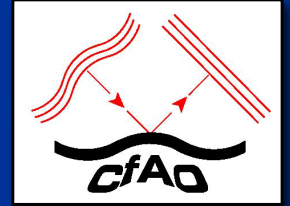
- “Seeing limited”: Units are radians
- Seeing disk gets slightly smaller at longer wavelengths:

$$FWHM \sim \lambda / \lambda^{-6/5} \sim \lambda^{-1/5}$$

- For completely uncompensated images, wavefront error

$$\sigma_{uncomp}^2 = 1.02 (D / r_0)^{5/3}$$

Correcting tip-tilt has relatively large effect, for seeing-limited images



- For completely uncompensated images

$$\sigma^2_{uncomp} = 1.02 (D / r_0)^{5/3}$$

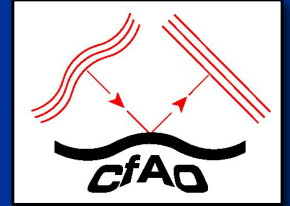
- If image motion (tip-tilt) has been completely removed

$$\sigma^2_{tiltcomp} = 0.134 (D / r_0)^{5/3}$$

(Tyson, Principles of AO, eqns 2.61 and 2.62)

- Removing image motion can (in principle) improve the wavefront variance of an uncompensated image by a factor of 10
- Origin of statement that “Tip-tilt is the single largest contributor to wavefront error”

But you have to be careful if you want to apply this statement to AO correction



- If tip-tilt has been completely removed

$$\sigma^2_{tiltcomp} = 0.134 (D / r_0)^{5/3}$$

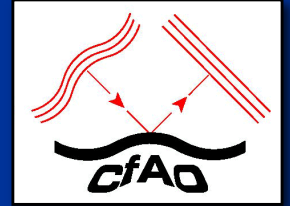
- But typical values of (D / r_0) are 10 - 50 in the near-IR
 - Keck, $D=10$ m, $r_0 = 60$ cm at $\lambda=2 \mu\text{m}$, $(D/r_0) = 17$

$$\sigma^2_{tiltcomp} = 0.134 (17)^{5/3} \sim 15$$

so wavefront phase variance is $\gg 1$

- Conclusion: if $(D/r_0) \gg 1$, removing tilt alone won't give you anywhere near a diffraction limited image

Scaling of image motion due to turbulence



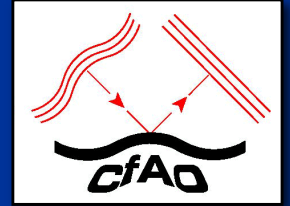
- Mean squared deflection angle due to image motion: independent of λ and $\propto D^{-1/3}$

$$\sigma_{\alpha}^2 = 0.182 \left(\frac{D}{r_0} \right)^{5/3} \left(\frac{\lambda}{D} \right)^2 \text{ radians}^2$$

- But relative to Airy disk (diffraction limit), image motion gets worse for larger D and smaller wavelengths:

$$\frac{\sigma_{\alpha}}{(\lambda / D)} = 0.43 \left(\frac{D}{r_0} \right)^{5/6} \propto \frac{D^{5/6}}{\lambda}$$

Typical values of image motion



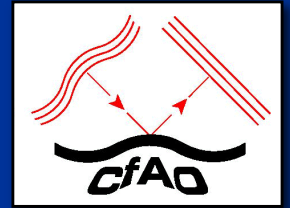
- Keck Telescope: $D = 10 \text{ m}$, $r_0 = 0.2 \text{ m}$, $\lambda = 2 \text{ microns}$

$$\sigma_{\alpha} = 0.43 \left(\frac{D}{r_0} \right)^{5/6} \left(\frac{\lambda}{D} \right) = 0.43 \left(\frac{10\text{m}}{0.2\text{m}} \right)^{5/6} \left(\frac{2 \times 10^{-6}\text{m}}{10\text{m}} \right) = 0.45 \text{ arc sec}$$

$$\frac{\lambda}{D} = 0.04 \text{ arc sec} \quad (\text{Recall that } 1 \text{ arcsec} = 5 \mu\text{rad})$$

- So in theory at least, rms image motion is > 10 times larger than diffraction limit, for these numbers.

What maximum tilt angle must the tip-tilt mirror be able to correct?



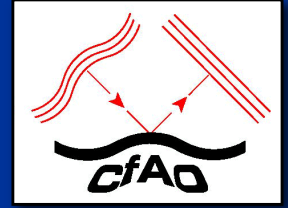
- For a Gaussian distribution, probability is 99.4% that the value will be within ± 2.5 standard deviations of the mean.
- For this condition, the peak excursion of the angle of arrival is

$$\alpha_{peak} = \pm 1.07 \left(\frac{D}{r_0} \right)^{5/6} \left(\frac{\lambda}{D} \right) \text{ radians} \approx 2 \text{ arc sec}$$

\uparrow
 2.5σ

- Note that peak angle is independent of wavelength

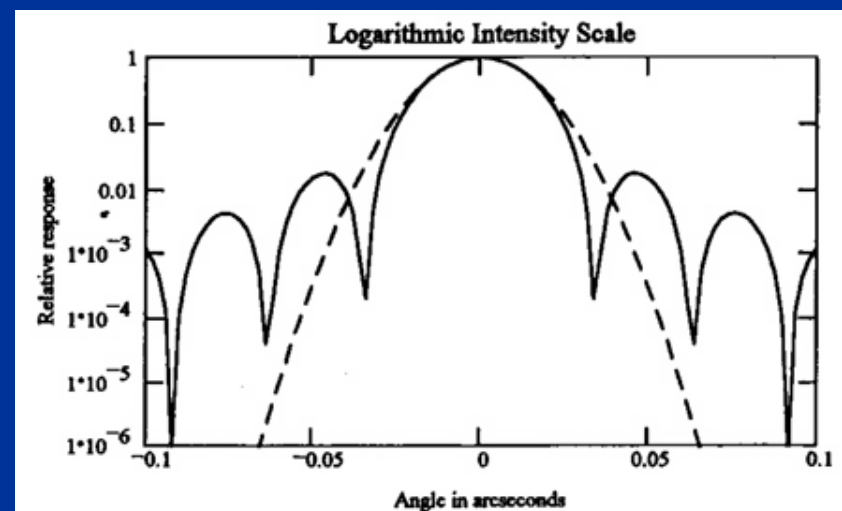
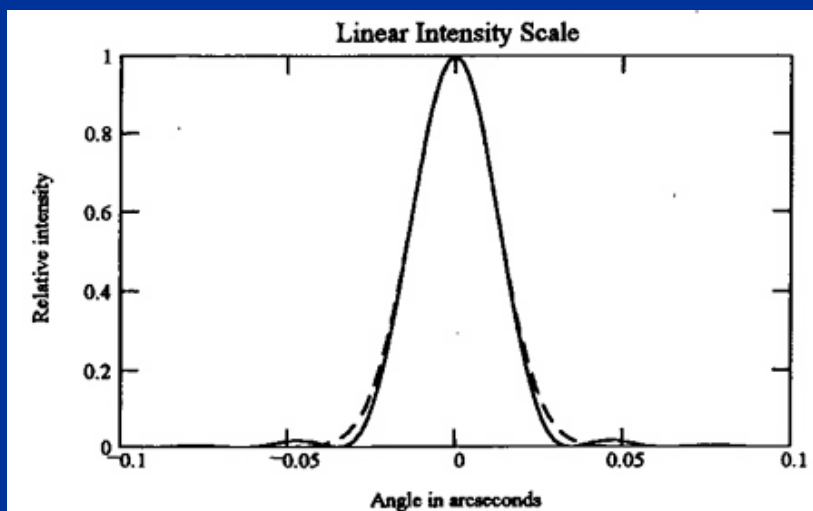
Use Gaussians to model the effects of image motion on image quality



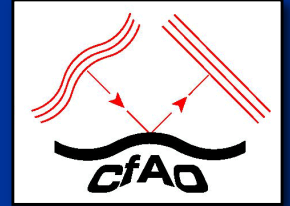
- Model the diffraction limited core as a Gaussian:

$$G(x) = \frac{1}{(2\pi)^{1/2}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

- A Gaussian profile with standard deviation $\sigma_A = 0.44 \left(\frac{\lambda}{D}\right)$ has same width as an Airy function



Tilt errors spread out the core



- Effect of random tilt error σ_α is to spread each point of image into a Gaussian profile with standard deviation σ_α
- If initial profile has width σ_A then the profile with tilt has width $\sigma_T = (\sigma_\alpha^2 + \sigma_A^2)^{1/2}$ (see next slide)

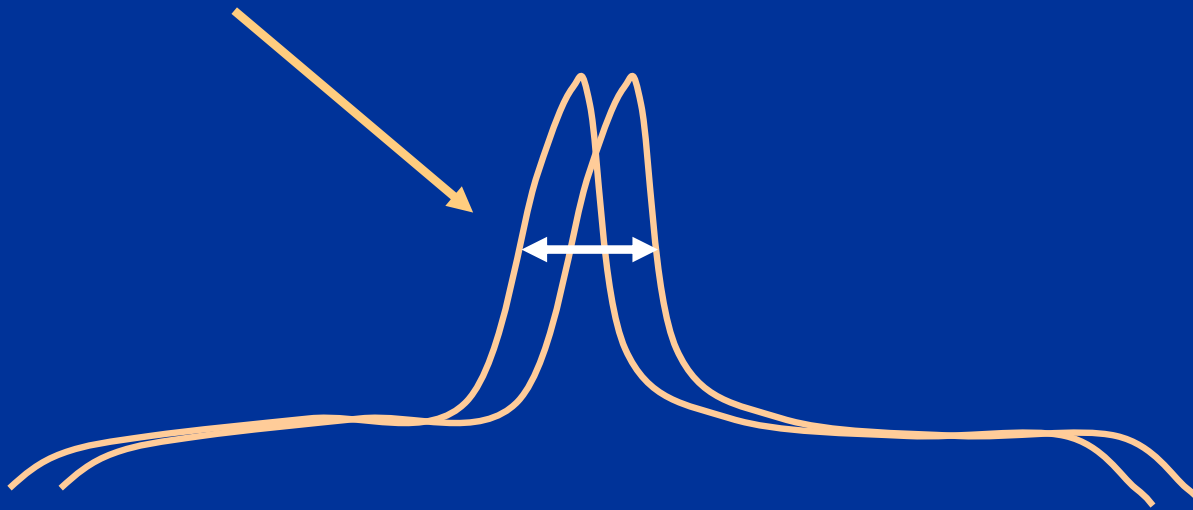
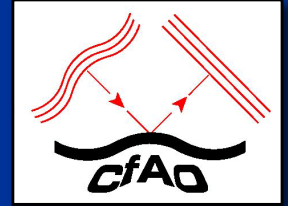


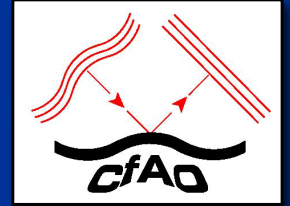
Image motion reduces peak intensity



- Conserve flux:
 - Integral under a circular Gaussian profile with peak amplitude A_0 is equal to $2\pi A_0 \sigma_A^2$
 - Image motion keeps total energy the same, but puts it in a new Gaussian with variance $\sigma_T^2 = \sigma_A^2 + \sigma_\alpha^2$
 - Peak intensity is reduced by the ratio

$$F_T = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_\alpha^2} = \frac{1}{1 + (\sigma_\alpha / \sigma_A)^2}$$

Tilt effects on point spread function, continued

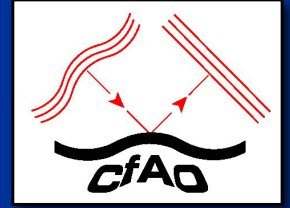


- Since $\sigma_A = 0.44 \lambda / D$, peak intensity of the previously diffraction-limited core is reduced by

$$F_T = \frac{1}{1 + (D / 0.44 \lambda)^2 \sigma_\alpha^2}$$

- Diameter of core is increased by $F_T^{-1/2}$
- Similar calculations for the halo: replace D by r_0
- Since $D \gg r_0$ for cases of interest, effect on halo is modest but effect on core can be large

Typical values for Keck Telescope, if tip-tilt is not corrected

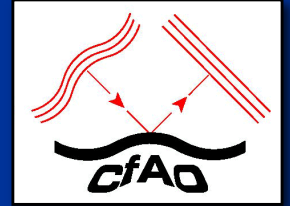


- Core is strongly affected at a wavelength of 1 micron:

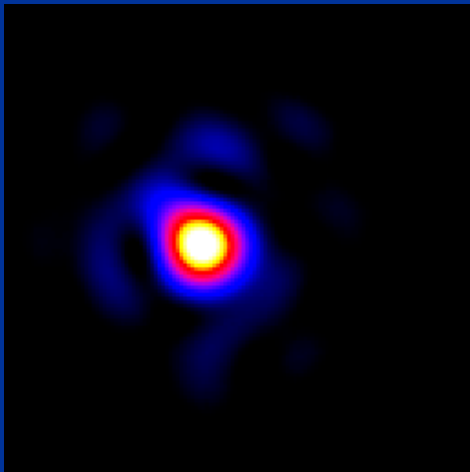
$$\sigma_{\alpha} \cong 0.5 \text{ arcsec}, \lambda / D = 0.02 \text{ arcsec}, F_T = \frac{1}{1 + (\sigma_{\alpha} / \sigma_A)^2} \approx 0.002$$

- Core diameter is increased by factor of $F_T^{-1/2} \sim 23$
- Halo is much less affected than core:
 - Halo peak intensity is only reduced by factor of 0.93
 - Halo diameter is only increased by factor of 1.04

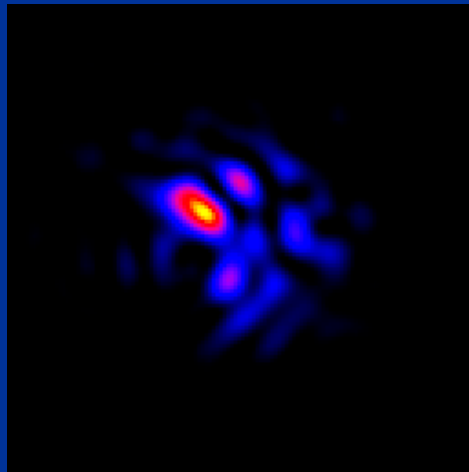
Effects of turbulence depend on size of telescope



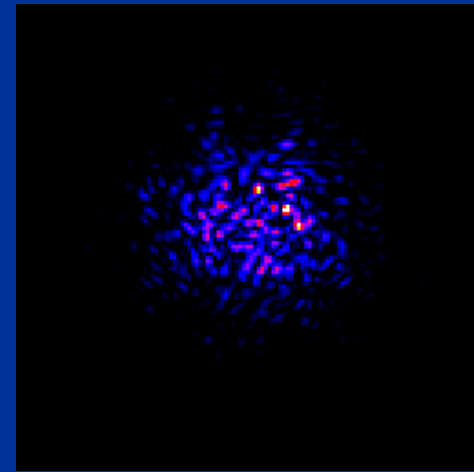
- Coherence length of turbulence: r_0 (Fried's parameter)
- For telescope diameter $D < (2 - 3) \times r_0$:
Dominant effect is "image wander"
- As D becomes $\gg r_0$:
Many small "speckles" develop
- Computer simulations by Nick Kaiser: image of a star, $r_0 = 40$ cm



$D = 1$ m

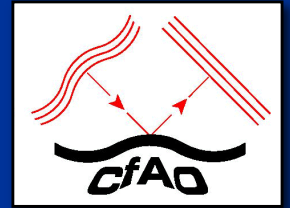


$D = 2$ m

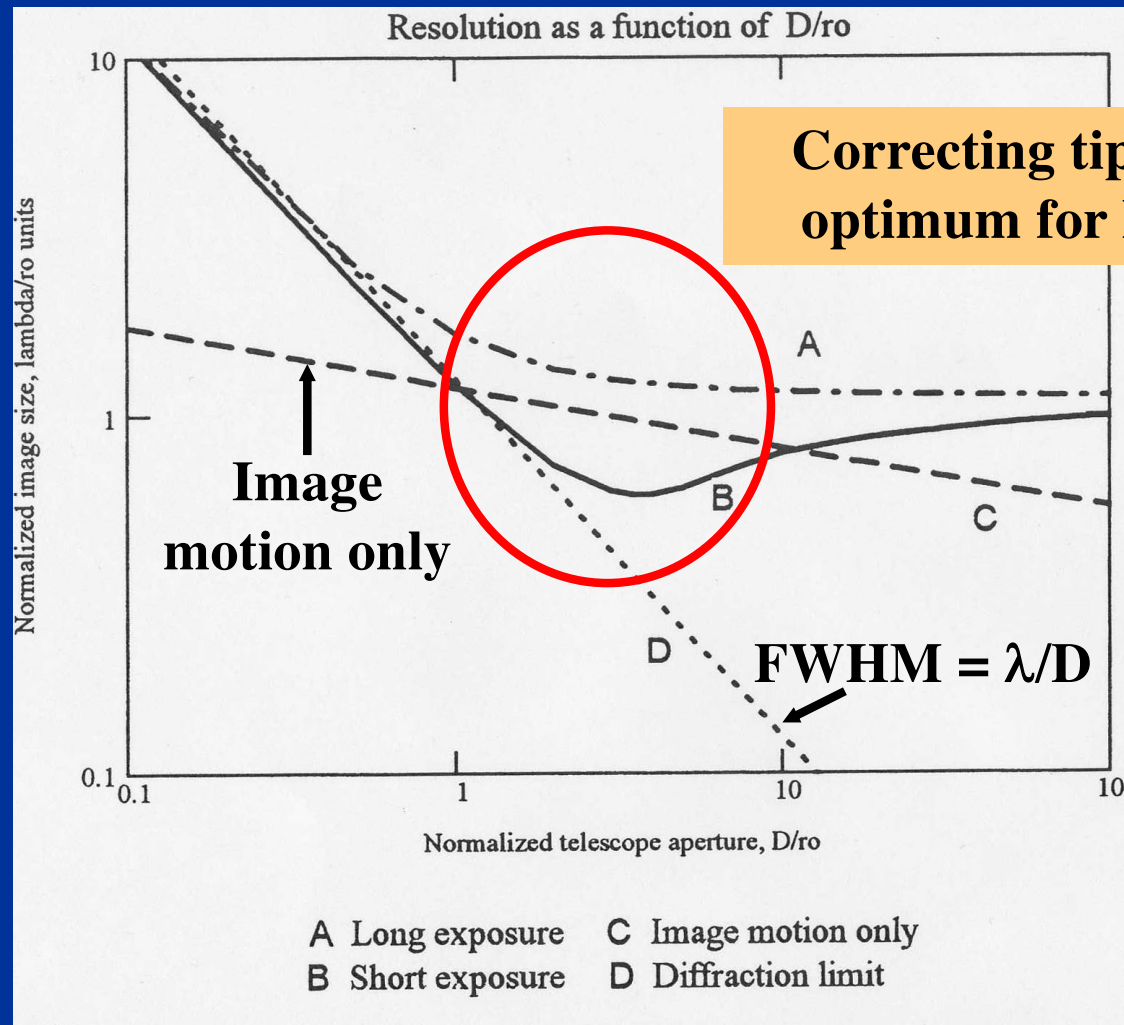


$D = 8$ m

Effect of atmosphere on long and short exposure images of a star



Hardy p. 94



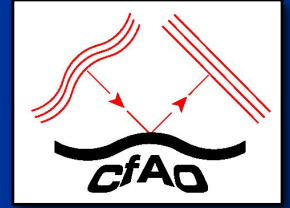
Vertical axis is image size in units of λ / r_0

Summary, Image Motion Effects (1)

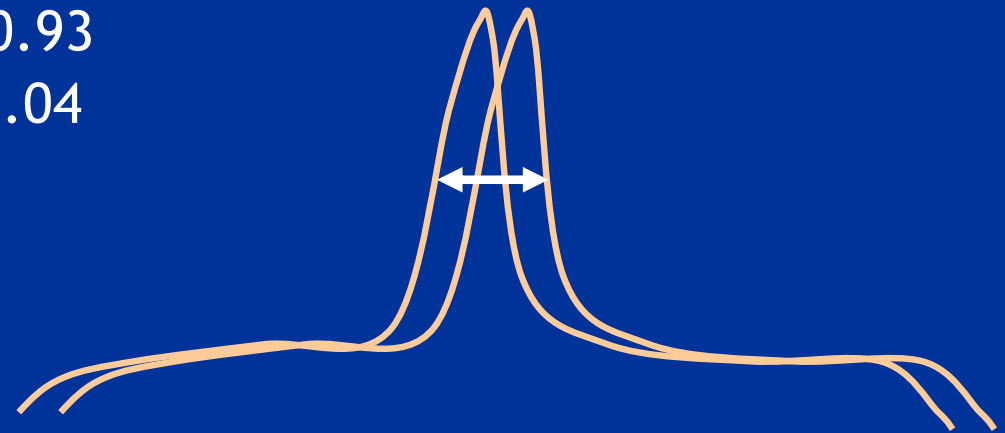


- Image motion
 - Broadens core of AO PSF
 - Contributes to Strehl degradation differently than high-order aberrations
 - Effect on Strehl ratio can be quite large: crucial to correct tip-tilt

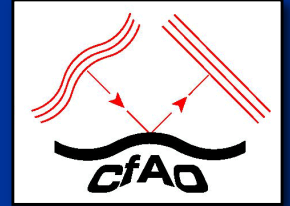
Summary, Image Motion Effects (2)



- Image motion can be large, if not compensated
 - Keck, $\lambda = 1$ micron, $\sigma_\alpha = 0.5$ arc sec
- Enters computation of overall Strehl ratio differently than higher order wavefront errors
- Lowers peak intensity of core by $F_c^{-1} \sim 1 / 0.002 = 500$ x
- Halo is much less affected:
 - Peak intensity decreased by 0.93
 - Halo diameter increased by 1.04

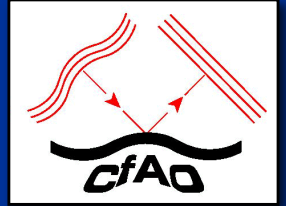


How to correct for image motion



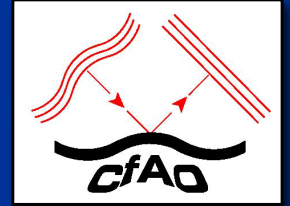
- Natural guide star AO:
 - From wavefront sensor information, filter out all higher order modes, left with overall tip-tilt
 - Correct this tip-tilt with a “tip-tilt mirror”
- Laser guide star AO:
 - Can use laser to correct for high-order aberrations but not for image motion (laser goes both up and down thru atmosphere, hence moves relative to stars)
 - So LGS AO needs to have a so-called “tip-tilt star” within roughly an arc min of target.
 - Can be faint: down to 18-19th magnitude will work

Implications of image motion for AO system design



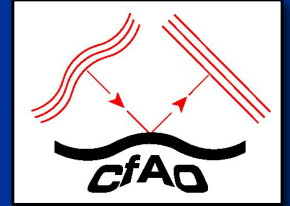
- Impact of image motion will be different, depending on the science you want to do
- Example 1: Search for planets around nearby stars
 - You can use the star itself for tip-tilt info
 - Little negative impact of image motion smearing
- Example 2: Studies of high-redshift galaxies
 - Sufficiently bright tip-tilt stars will be rare
 - Trade-off between fraction of sky where you can get adequate tip-tilt correction, and the amount of tolerable image-motion blurring
 - » High sky coverage → fainter tip-tilt stars farther away

Part 2: Detectors and signal to noise ratio



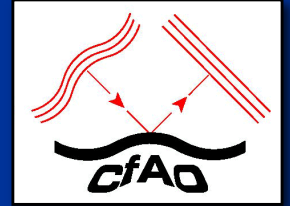
- Detector technology
 - Basic detector concepts
 - Modern detectors: CCDs and IR arrays
- Signal-to-Noise Ratio (SNR)
 - Introduction to noise sources
 - Expressions for signal-to-noise
 - » Terminology is not standardized
 - » Two Keys: 1) Write out what you're measuring.
2) Be careful about units!
 - » Work directly in photo-electrons where possible

References for detectors and signal to noise ratio

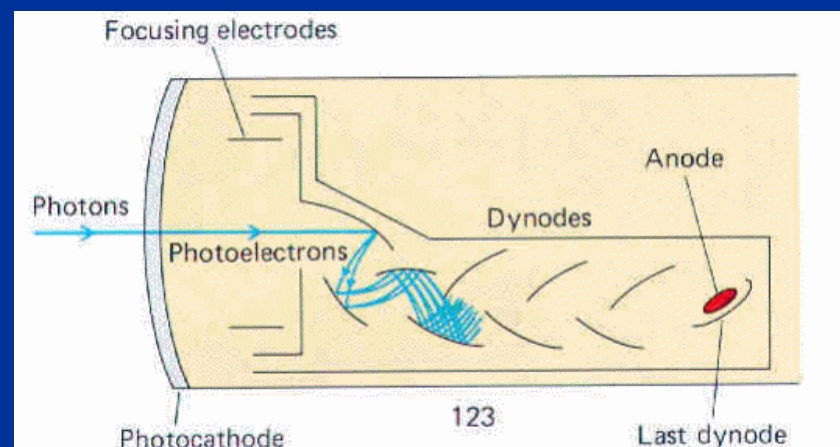


- Excerpt from “Electronic imaging in astronomy”, Ian. S. McLean (1997 Wiley/Praxis)
- Excerpt from “Astronomy Methods”, Hale Bradt (Cambridge University Press)

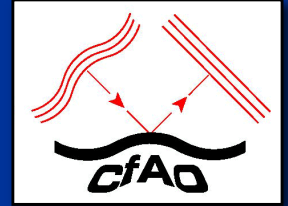
Early detectors: photographic plates, photomultipliers



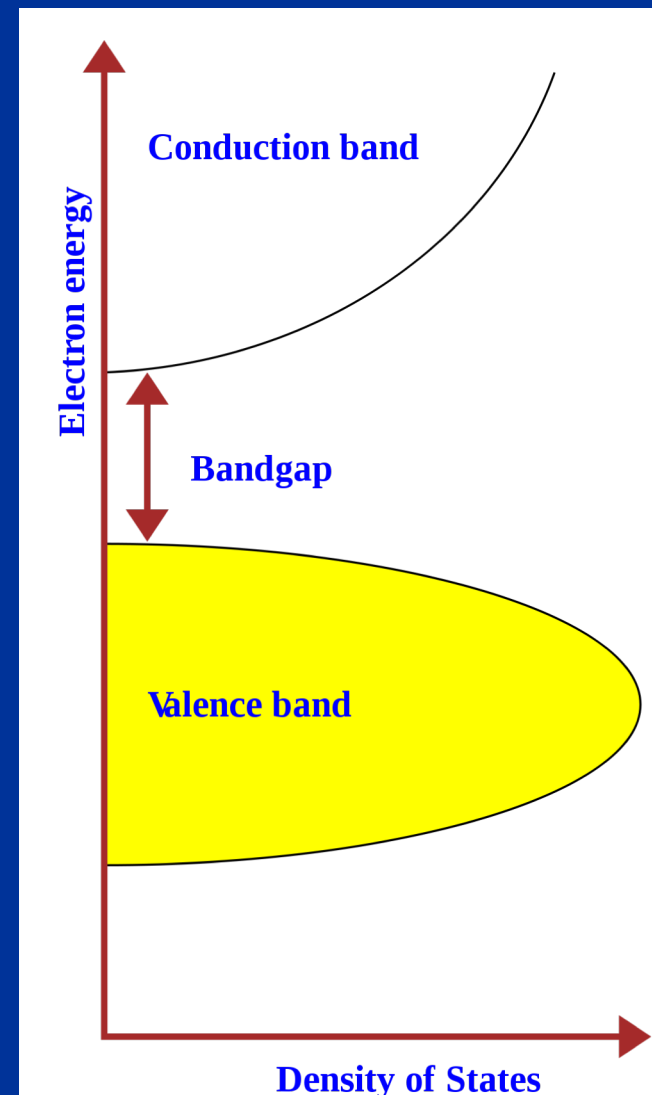
- Photographic plates
 - very low Quantum Efficiency: QE ~ 1 - 4%
 - non-linear response
 - very large areas, very small “pixels” (grains of silver compounds)
 - hard to digitize
- Photomultiplier tubes
 - low QE (10%)
 - no noise: each photon produces cascade
 - linear at low signal rates
 - easily coupled to digital outputs



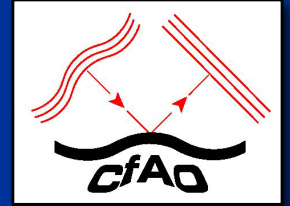
Modern detectors are based on semiconductors



- In semiconductors and insulators, electrons are confined to a number of specific bands of energy
- “Band gap” = energy difference between top of valence band and bottom of the conduction band
- For an electron to jump from a valence band to a conduction band, need a minimum amount of energy
- This energy can be provided by a photon, or by thermal energy, or by a cosmic ray
- Vacancies or holes left in valence band allow it to contribute to electrical conductivity as well
- Once in conduction band, electron can move freely and be collected



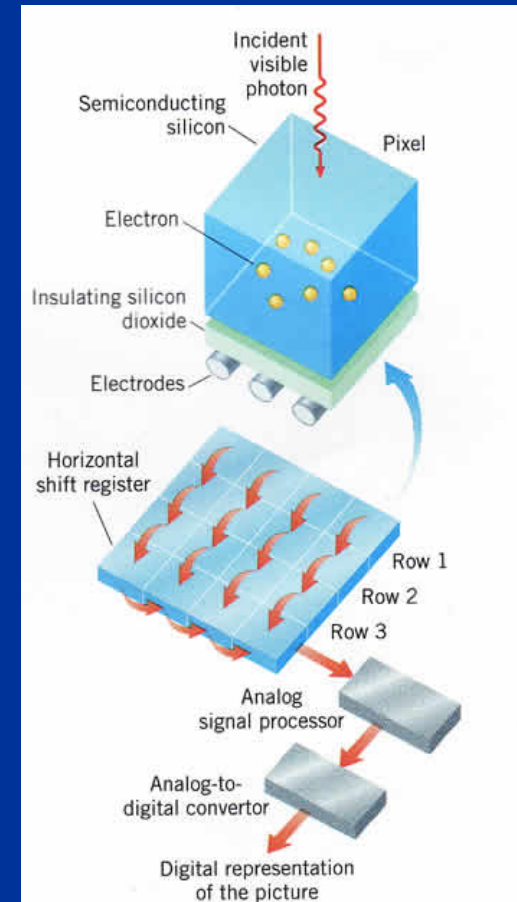
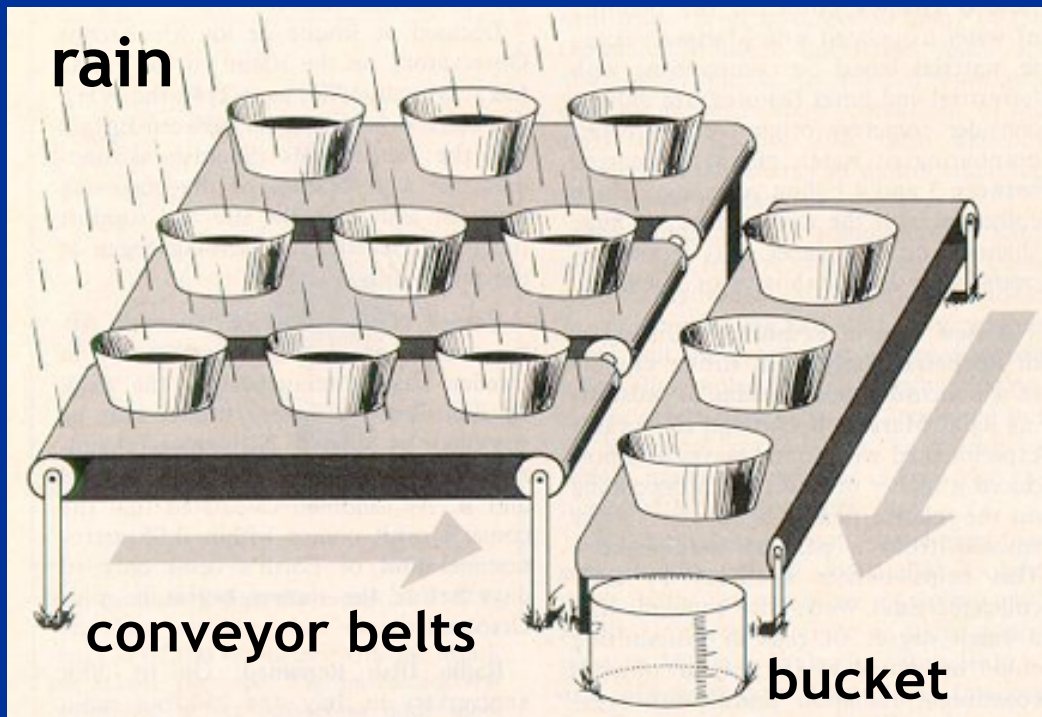
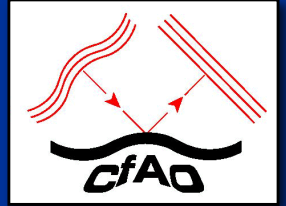
Bandgap energies for commonly used detectors



- If the forbidden energy gap is E_G there is a cut-off wavelength beyond which the photon energy (hc/λ) is too small to cause an electron to jump from the valence band to the conduction band

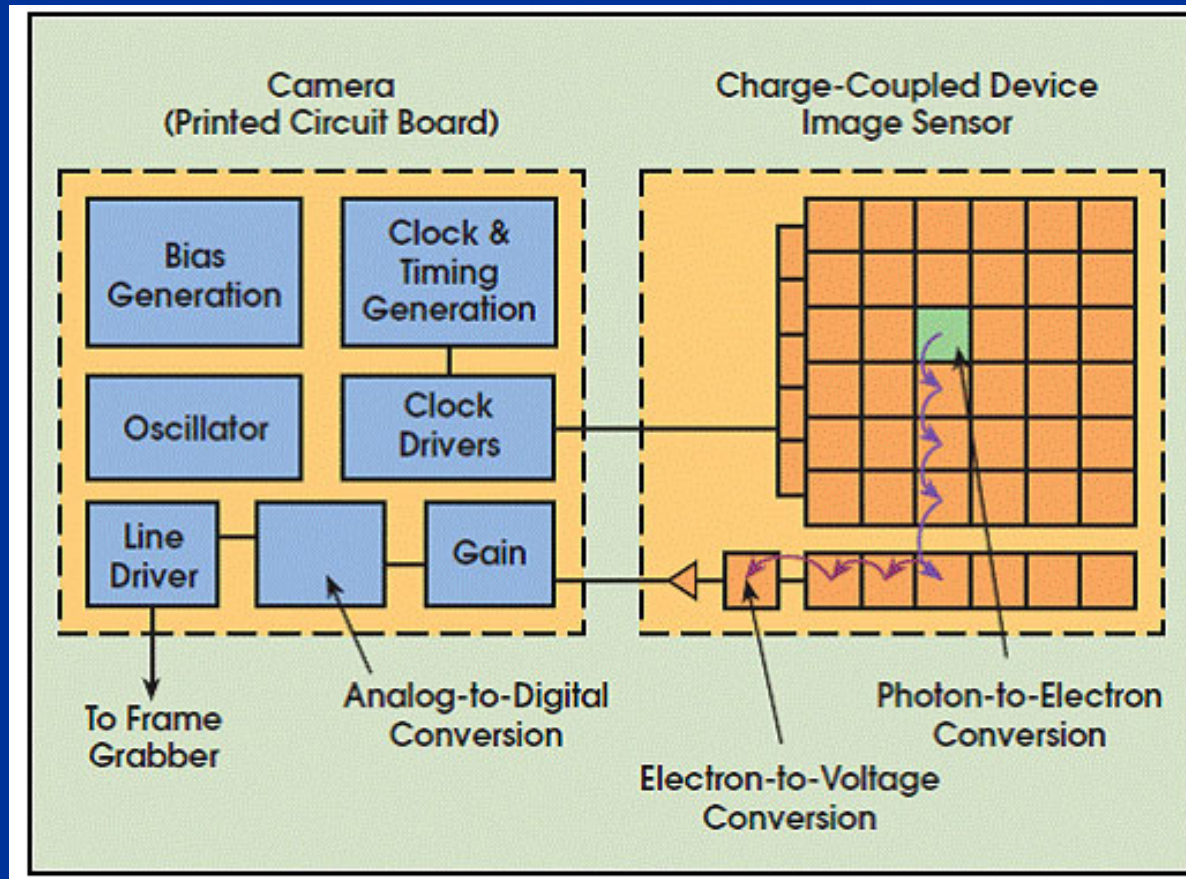
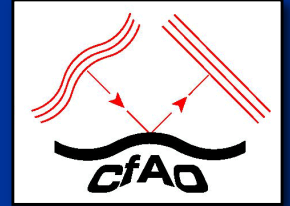
Material	Symbol	E_{gap}	λ_{cutoff}
Silicon	Si	1.12 eV	1.1 μm
Indium antimonide	InSb	0.23 eV @ 77K	5.4 μm
Mercury cadmium telluride	$\text{Hg}_x\text{Cd}_{1-x}\text{Te}$	0.5 eV ($x=0.55$)	2.5 μm

CCD transfers charge from one pixel to the next in order to make a 2D image



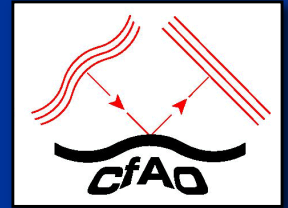
- By applying “clock voltage” to pixels in sequence, can move charge to an amplifier and then off the chip

Schematic of CCD and its read-out electronics

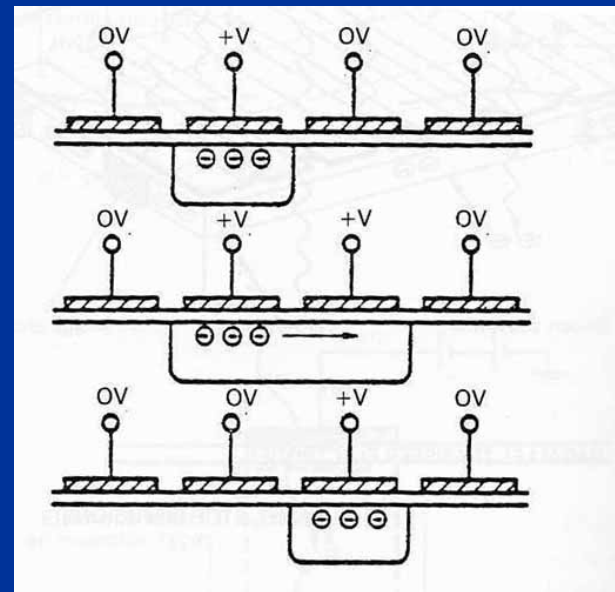


- “Read-out noise” injected at the on-chip electron-to-voltage conversion (an on-chip amplifier)

CCD readout process: charge transfer



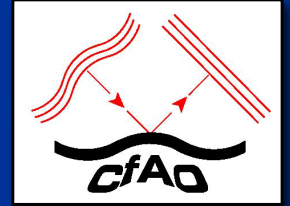
- Adjusting voltages on electrodes connects wells and allow charge to move
- Charge shuffles up columns of the CCD and then is read out along the top
- Charge on output amplifier (capacitor) produces voltage



Video animation:

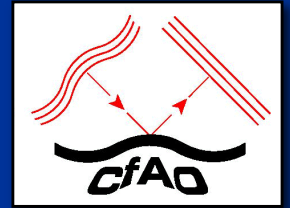
<https://www.youtube.com/watch?v=PoXinWleWns>

*Modern detectors: photons → electrons
→ voltage → digital numbers*



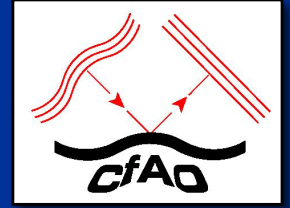
- With what efficiency do photons produce electrons?
- With what efficiency are electrons (voltages) measured?
- Digitization: how are electrons (analog) converted into digital numbers?
- Overall: What is the conversion between photons hitting the detector and digital numbers read into your computer?

Primary properties of detectors



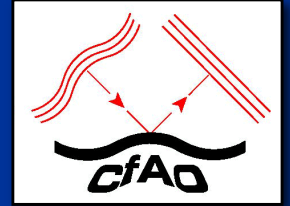
- Quantum Efficiency QE : Probability of detecting a single photon incident on the detector
- Spectral range (QE as a function of wavelength)
- “Dark Current”: Detector signal in the absence of light
- “Read noise”: Random variations in output signal when you read out a detector
- Gain g : Conversion factor between internal voltages and computer “Data Numbers” DNs or “Analog-to-Digital Units” ADUs

Secondary detector characteristics



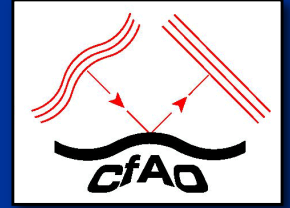
- Pixel size (e.g. in microns)
- Total detector size (e.g. 1024 x 1024 pixels)
- Readout rate (in either frames per sec or pixels per sec)
- Well depth (the maximum number of photons or photoelectrons that a pixel can record without “saturating” or going nonlinear)
- Cosmetic quality: Uniformity of response across pixels, dead pixels
- Stability: does the pixel response vary with time?

CCDs are the most common detector for wavefront sensors



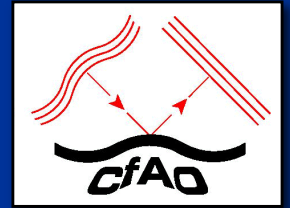
- Can be read out fast (e.g., every few milliseconds so as to keep up with atmospheric turbulence)
- Relatively low read-noise (a few to 10 electrons)
- Only need modest size (e.g., largest today is only 256x256 pixels)

CCD phase space

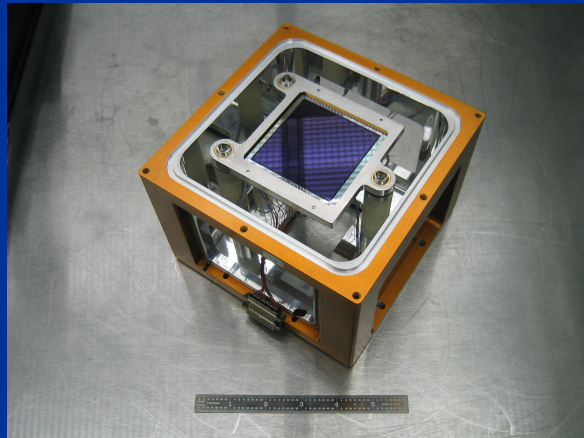


- CCDs dominate inside and outside astronomy
 - Even used for x-rays
- Large formats available (4096x4096) or mosaics of smaller devices. Gigapixel focal planes exist.
- High quantum efficiency 80%+
- Dark current from thermal processes
 - Long-exposure astronomy CCDs are cooled to reduce dark current
- Readout noise can be several electrons per pixel each time a CCD is read out
 - » Trade high readout speed vs added noise
 - » To reduce read noise, usually have to reduce speed

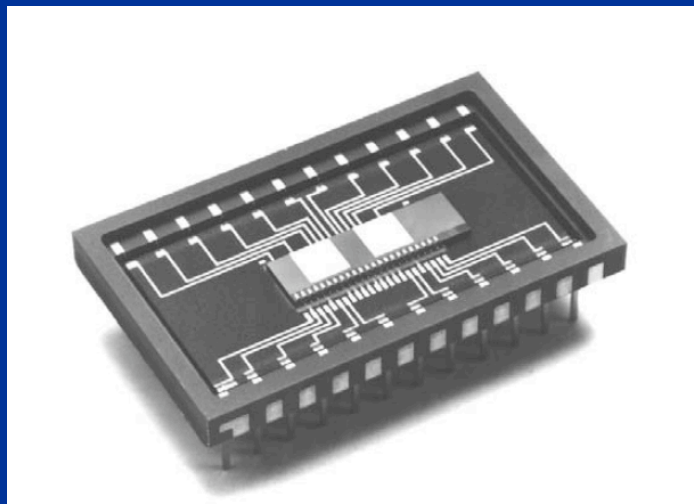
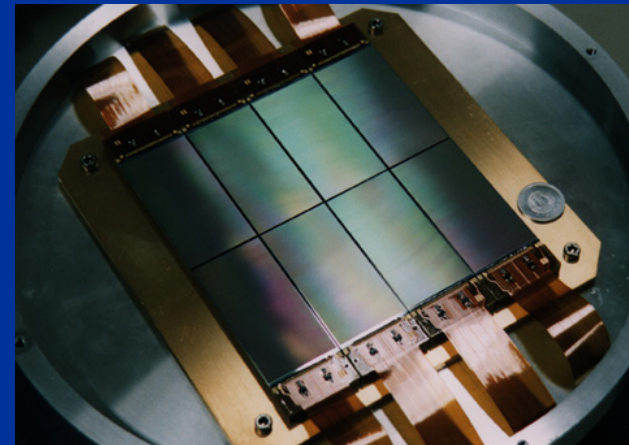
What do CCDs look like?



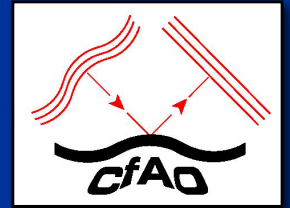
Carnegie 4096x4096 CCD
Slow readout (science)



Subaru SuprimeCam Mosaic
Slow readout (science)

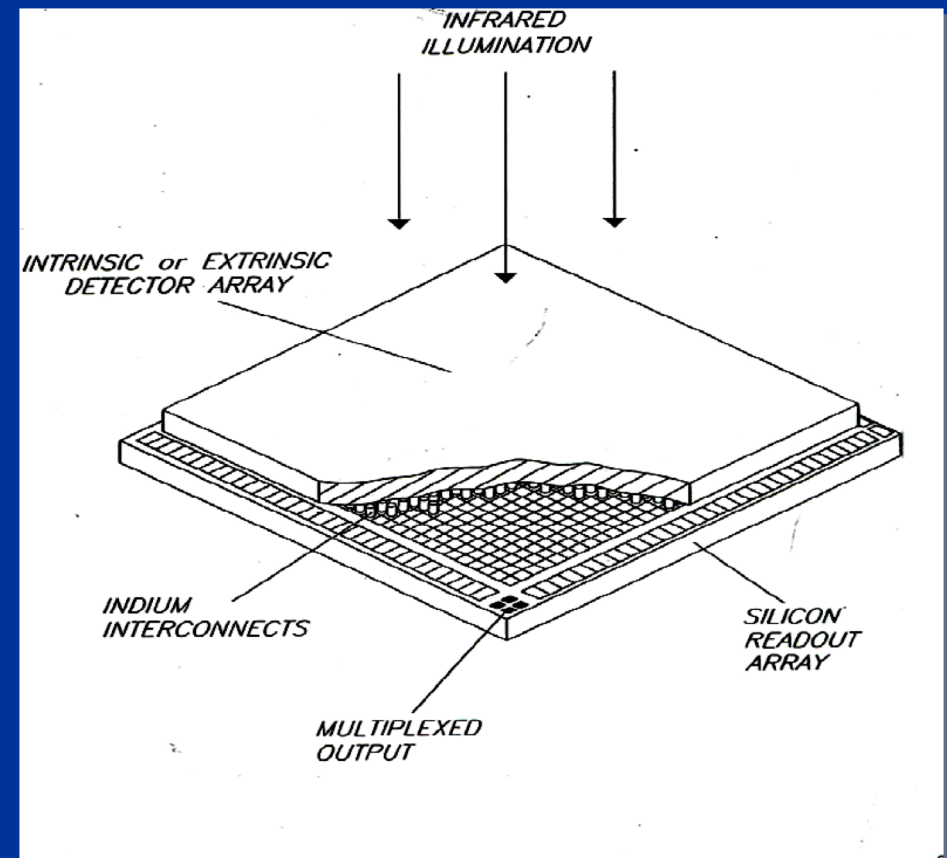


E2V 80 x 80
fast readout for
wavefront sensing

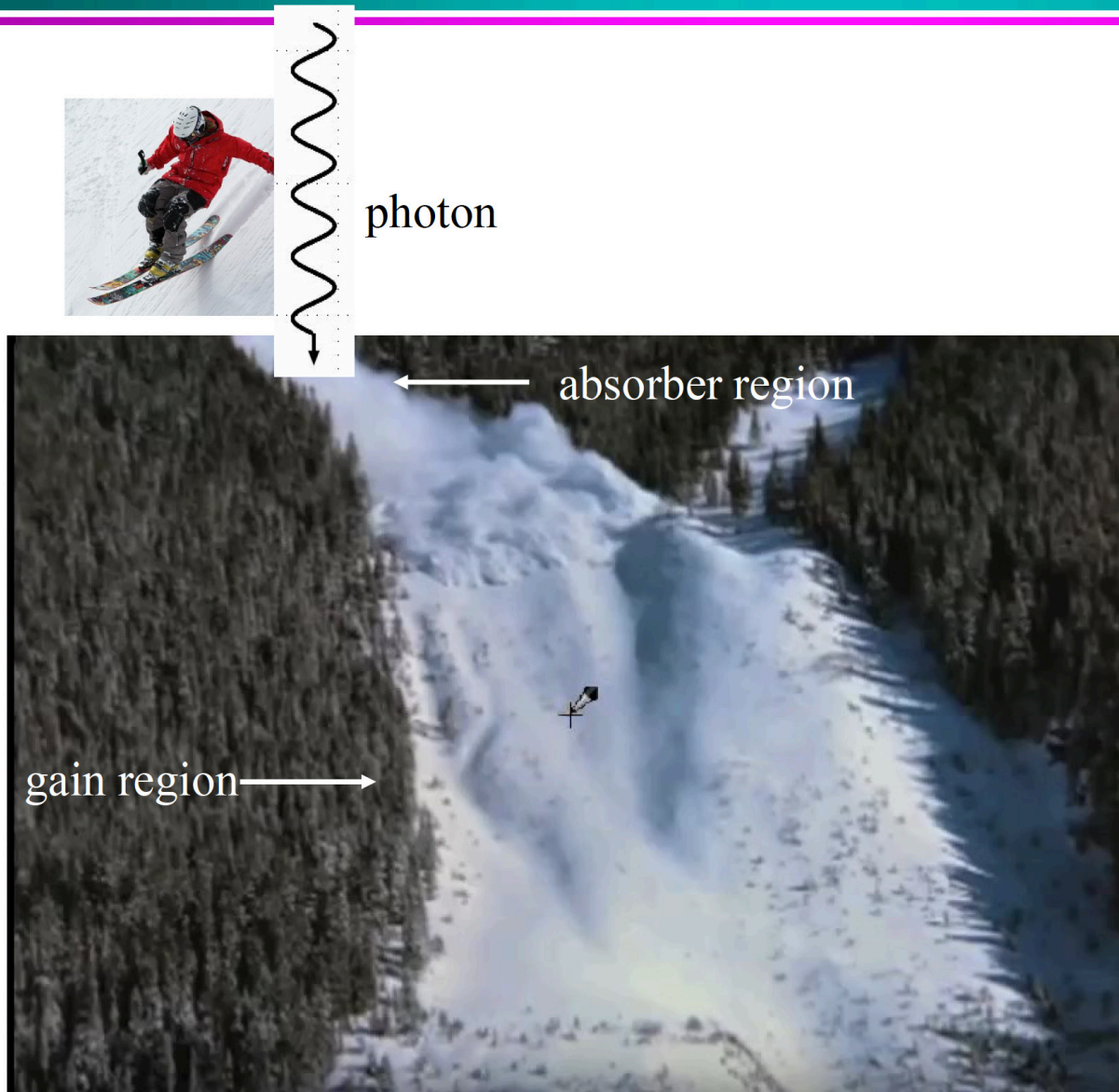


Infrared detectors (e.g. Hawaii 2RG)

- Read out pixels individually, by bonding a multiplexed readout array to the back of the photo-sensitive material
- Photosensitive material must have lower band-gap than silicon, in order to respond to lower-energy IR photons
- Materials: InSb, HgCdTe, ...



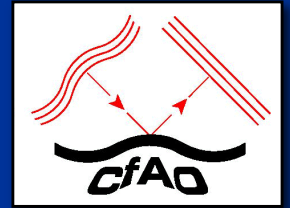
electron Avalanche PhotoDiode : eAPD



- absorbed photon creates free electron
- electron accelerated in electric field
- by impact ionization an avalanche of electrons is created
- it is easier to detect an avalanche of electrons than a single electron

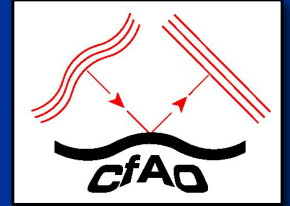
Credit: Gert Finger

Recent development: IR Avalanche Photodiode Arrays -- Saphira



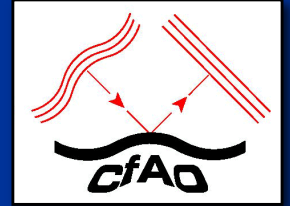
- Until now we have not had high frame rate, low noise infrared detector arrays, so it was not feasible to do high-order wavefront sensing at near-infrared wavelengths.
- Selex Avalanche Photodiode for HgCdTe InfraRed Array (SAPHIRA) detector is the first technology that enables this
- SAPHIRA detectors are HgCdTe linear avalanche photodiode arrays
- 320×256 pixels (enough for wavefront sensing at least on 8-10m telescopes)
- Sensitive to 0.8-2.5 μm light.
- Key features:
 - Fast readout rates while maintaining low readout noise
 - Low dark current
 - Ability to read out subarrays without a noise penalty
- Ideal for infra-red wavefront sensing

Types of noise in instruments



- Every instrument has its own characteristic background noise
 - Example: cosmic ray particles passing thru a CCD knock electrons into the conduction band
- Some residual instrument noise is **statistical** in nature; can be measured very well given enough integration time
- Some residual instrument noise is **systematic** in nature: cannot easily be eliminated by better measurement
 - Example: difference in optical path to wavefront sensor and to science camera
 - Typically has to be removed via **calibration**

Statistical fluctuations = “noise”



- Definition of variance:

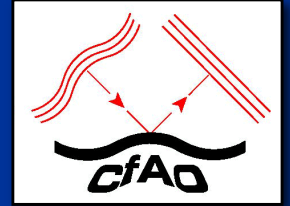
$$\sigma^2 \equiv \frac{1}{n} \sum_{i=1}^n (x_i - m)^2$$

where m is the mean, n is the number of independent measurements of x , and the x_i are the individual measured values

- If x and y are two random independent variables, the variance of the sum (or difference) is the sum of the variances:

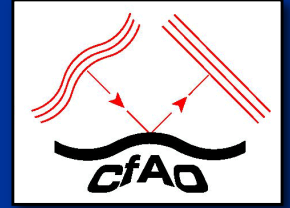
$$\sigma_{tot}^2 = \sigma_x^2 + \sigma_y^2$$

Main sources of detector noise for wavefront sensors in common use



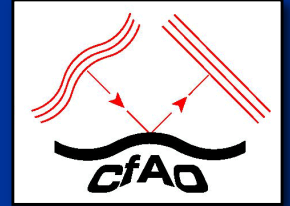
- Poisson noise or photon statistics
 - Noise due to statistics of the detected photons themselves
- Read-noise
 - Electronic noise (from amplifiers) each time CCD is read out
- Other noise sources (less important for wavefront sensors, but important for other imaging applications)
 - Sky background
 - Dark current

Photon statistics: Poisson distribution



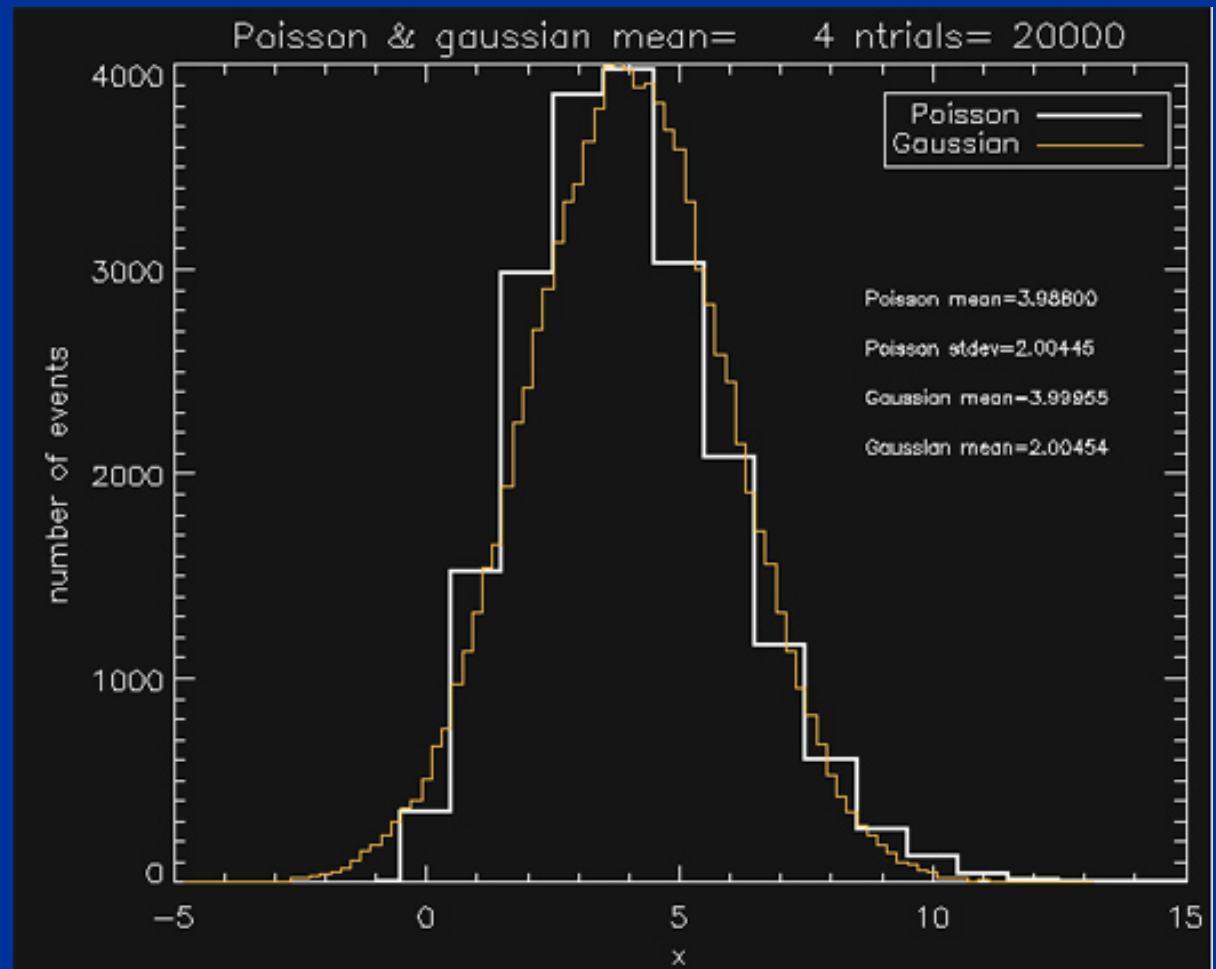
- CCDs are sensitive enough that they care about individual photons
- Light is quantum in nature. There is a natural variability in how many photons will arrive in a specific time interval T , even when the average flux F (photons/sec) is fixed.
- We can't assume that in a given pixel, for two consecutive observations of length T , the same number of photons will be counted.
- The probability distribution for N photons to be counted in an observation time T is

$$P(N|F, T) = \frac{(FT)^N e^{-FT}}{N!}$$



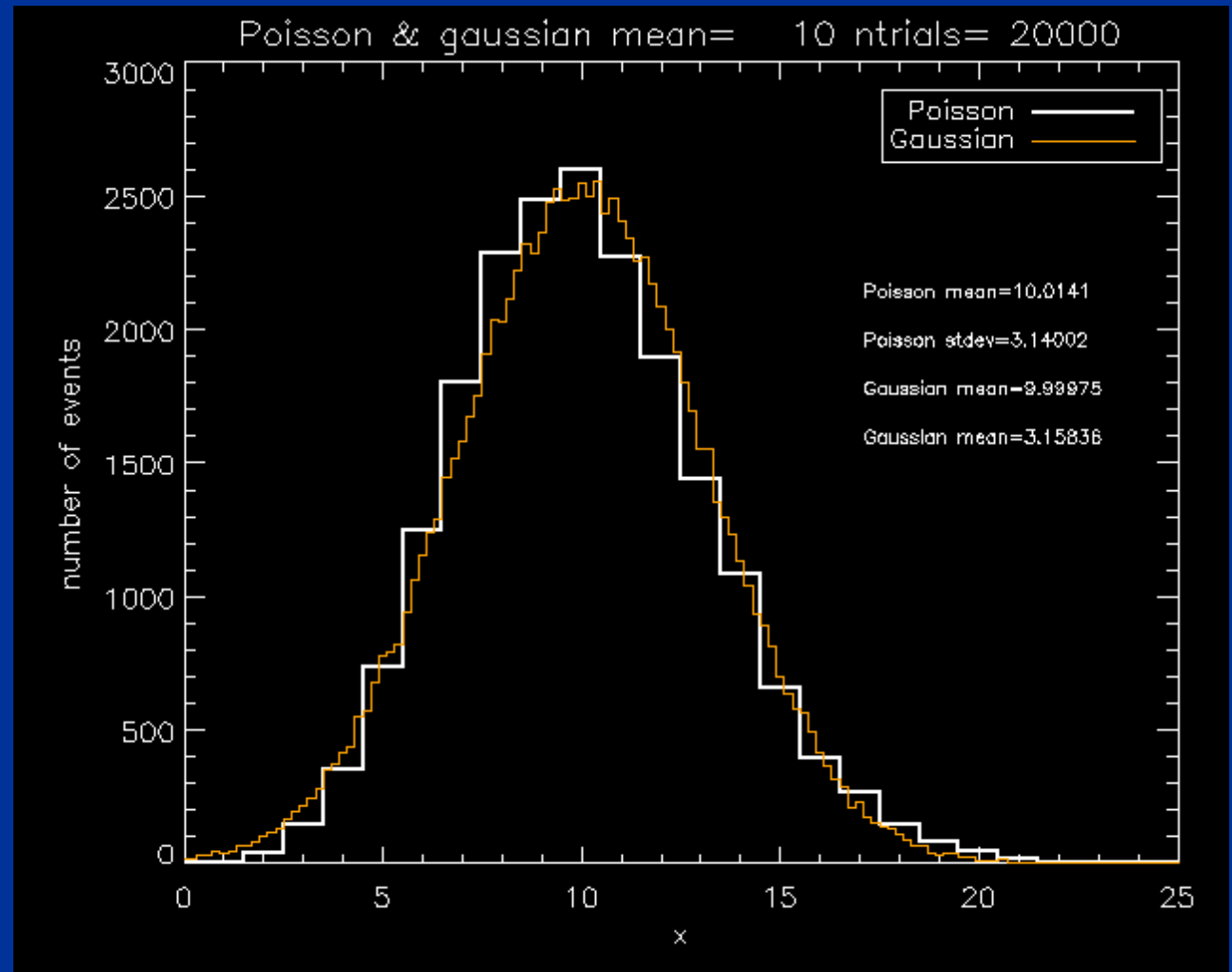
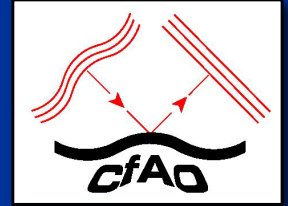
Properties of Poisson distribution

- Average value = FT
- Standard deviation = $(FT)^{1/2}$
- Approaches a Gaussian distribution as N becomes large

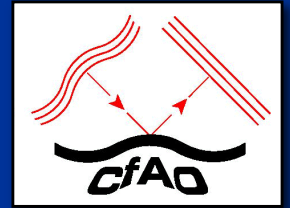


Horizontal axis: FT

Properties of Poisson distribution

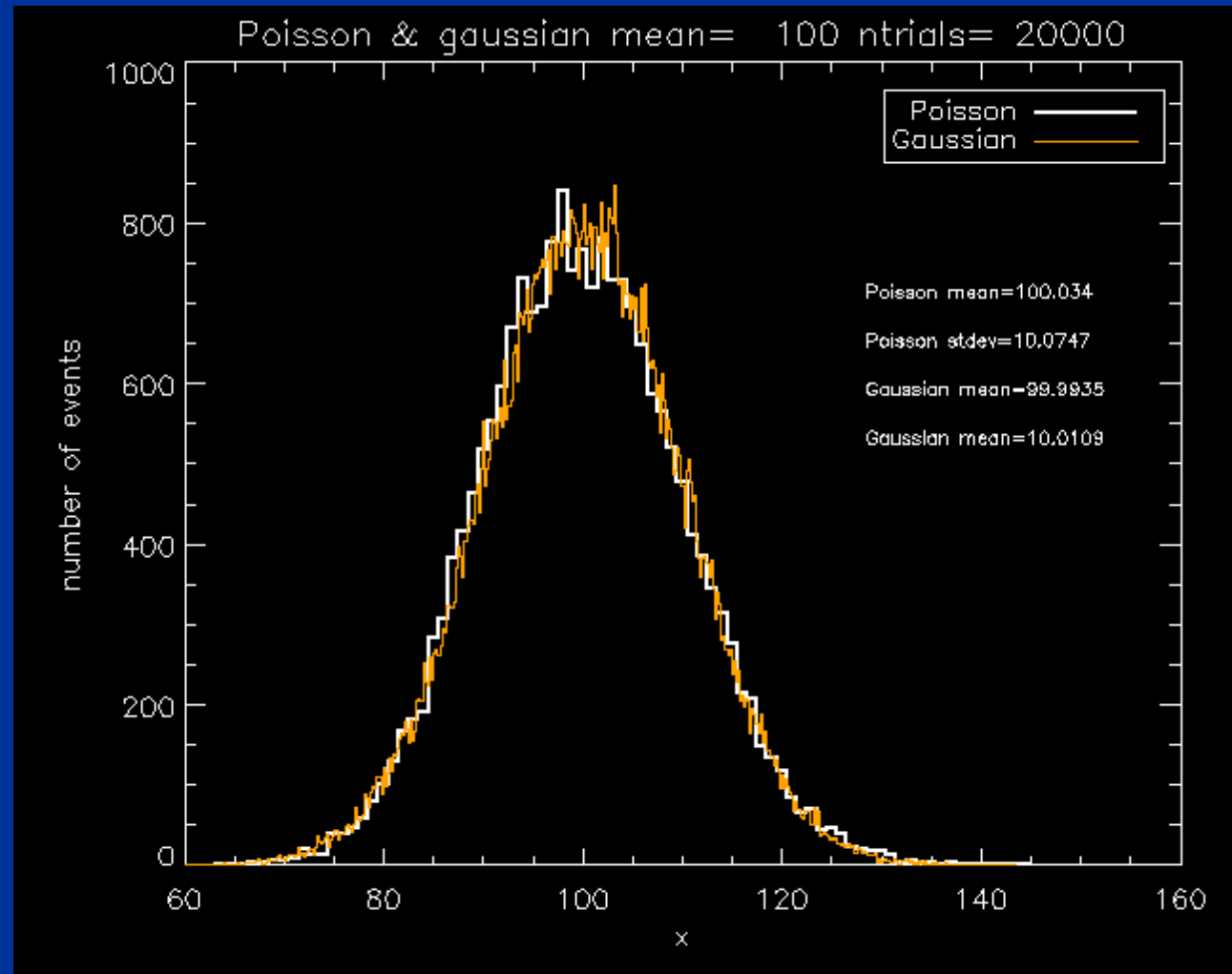


Horizontal axis: *FT*



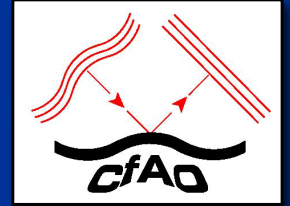
Properties of Poisson distribution

- When $\langle FT \rangle$ is large, Poisson distribution approaches Gaussian
- Standard deviations of independent Poisson and Gaussian processes can be added in quadrature



Horizontal axis: FT

How to convert between incident photons and recorded digital numbers ?

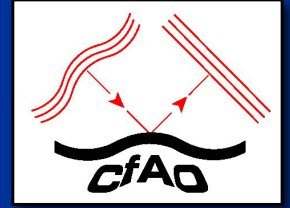


- Digital numbers outputted from a CCD are called Data Numbers (DN) or Analog-Digital Units (ADUs)
- Have to turn DN or ADUs back into microvolts, electrons, photons to have a calibrated system

$$\text{Signal in DN or ADU} = \left(\frac{QE \times N_{\text{photons}}}{g} \right) + b$$

where QE is the quantum efficiency (what fraction of incident photons get made into electrons), g is the photon transfer gain factor (electrons/DN) and b is an electrical offset signal or bias

To calculate SNR, look at all the various noise sources



- Wisest to calculate SNR in electrons rather than ADU or magnitudes
- Noise comes from Poisson noise in the object, Gaussian-like readout noise RN per pixel, Poisson noise in the sky background, and dark current noise D

- Readout noise: $\sigma_{RN}^2 = n_{pix} RN^2$

where n_{pix} is the number of pixels and RN is the readout noise

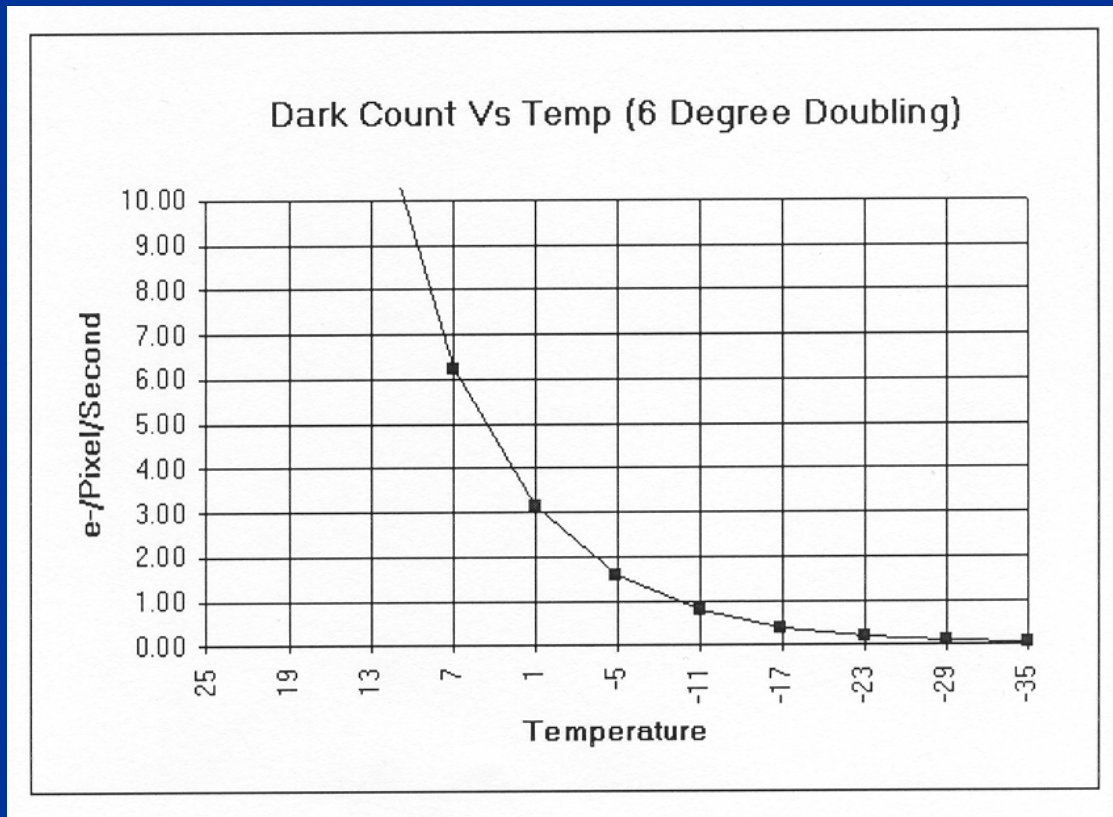
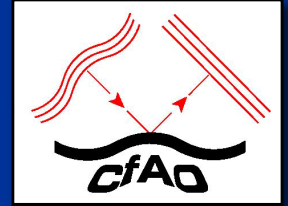
- Photon noise: $\sigma_{Poisson}^2 = FT = N_{photo-electrons}$

- Sky background: for B_{sky} e-/pix/sec from the sky, $\sigma_{Sky}^2 = B_{sky} T$

- Dark current noise: for dark current D (e-/pix/sec)

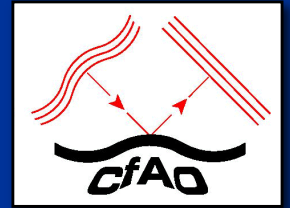
$$\sigma_{Dark}^2 = D n_{pix} T$$

Dark Current or Thermal Noise: Electrons reach conduction bands due to thermal excitation



Science CCDs are always cooled (liquid nitrogen, dewar, etc.)

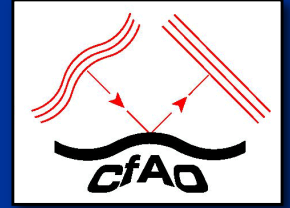
Total signal to noise ratio - important!



$$SNR = \frac{FT}{\sigma_{tot}} = \frac{FT}{\left[FT + (B_{sky}n_{pix}T) + (Dn_{pix}T) + (RN^2n_{pix}) \right]^{1/2}}$$

where F is the average photo-electron flux, T is the time interval of the measurement, B_{sky} is the electrons per pixel per sec from the sky background, D is the electrons per pixel per sec due to dark current, and RN is the readout noise per pixel.

Some special cases



- **Poisson statistics:** If detector has very low read noise, sky background is low, dark current is low, SNR is

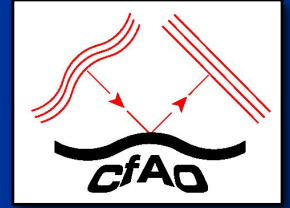
$$SNR_{Poisson} = \frac{FT}{\sqrt{FT}} = \sqrt{FT} \propto \sqrt{T}$$

- **Read-noise dominated:** If there are lots of photons but read noise is high, SNR is

$$SNR_{RN} = \frac{FT}{[RN^2 n_{pix}]^{1/2}} = \frac{FT}{RN \sqrt{n_{pix}}} \propto T$$

If you add multiple images, $SNR \sim (N_{images})^{1/2}$

Typical noise cases for astronomical AO



- Wavefront sensors

- Read-noise dominated: $SNR_{RN} = \frac{FT}{RN \sqrt{n_{pix}}}$

- Imagers (cameras)

- Sky background limited: $SNR_B = \frac{FT}{[B_{sky} n_{pix} T]^{1/2}} = \frac{F\sqrt{T}}{[B_{sky} n_{pix}]^{1/2}}$

- Spectrographs

- Either sky background or dark current limited:

$$SNR_B = \frac{F\sqrt{T}}{[B_{sky} n_{pix}]^{1/2}} \quad \text{or} \quad SNR_D = \frac{F\sqrt{T}}{[D n_{pix}]^{1/2}}$$